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## **ARTICULATION OF HIERARCHY AND NETWORKS AS AN EVOLVING SOCIAL STRUCTURE**

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# 1 Aim of the research and context

This paper describes simulations in an agent-based (or multi-agent) system, where we wish to generate an evolution of networks following different logics of communication for agents. The model is inspired by sociological observations led in a courthouse, in Paris, and by assumptions on the motivations that lead individuals to interact in this context. Two main ways to choose communication partner are here represented: following a pre-existing hierarchy with its own dynamics; inspired by past interactions. Both logics are described as well as their effect on the evolution of networks, and their combination is analyzed.

The study of networks has been of high importance for social studies since some seminal papers (White et al., 1976; Granovetter, 1973; Granovetter, 1985). Since then, the type and shape of network have been shown to be of great importance for information circulation, reciprocal obligation, and task success. One important notion that is studied in this respect is the one of centrality of an individual in a network (the number of ties that links him or her to others). Depending on how the global structure is organised, it is not the only important data to study. Indeed, for example in an organisation where information circulation is of central importance for activities, individuals can acquire power just by being the only one linking two separated groups (Burt, 1992). A purely individual-based approach to network is indeed impossible. A broader view of the shape of a network has to be defined (through ideas of density or cliquishness; or of structural equivalence) so as to study the global effect on the type of power relations, the global behaviour for the organisation, its performance (Lazega and Favereau, 2002; [McFadzean and Tesfatsion](#), 1999).

Adding to this static approach, some people emphasized diverse self-reinforcing dynamics for networks, and observed the evolution of ties in the system. Often, the approach relies on pay-off structures and anticipatory abilities or learning processes that influence the agents' links (Cowan et al. 2003; Deroian, 2003). This could lead to the study of pure structural characteristics, where an initial situation and the definition of links evolution could link to certain new shapes, and avoid the apparition of others (Jackson and Watts, 2002). Both static and dynamical approaches can be interesting to study structural characteristics. They can have an analytic orientation that rely more on economic issues (performance, efficiency, innovation dissemination) or more sociological issues (social processes such as solidarity, control, regulation, learning, etc.). Our research is orientated towards the representation of dynamical processes. The set of data is made of several static views of the system acquired through surveys led every two years (Lazega and Mounier, 2003). With the next step, it will end up giving some indication about the evolution of the system. The sociological approach that two of the authors enriched makes us stay away from most economic assumptions about learning in human interactions and from most economic indicators about a system.

One important issue in the study of networks is to show the difference between network-based dynamics and hierarchy-based dynamics (Marshack and Reichelstein; 1995, Thorelli, 1995). Very often they are considered to be opposed systems: for example one can study which form is most efficient between both for an organisation; one can try to build a hierarchy "from" a network using special payoffs (Slikker and al., 2005). In some previous work, one of the authors had also studied the issue of emerging hierarchy through interactions, where all agents were similar at the beginning and all linked together (Rouchier et al., 2000). Little analytical work, however, focuses on the interconnections between a pre-existing hierarchy and evolving networks, and the influence of both on the pattern of relations that emerges over time. The assumption here is that the relationships that are established among individuals (judges) in the Courthouse is influenced by a pre-existing formal structure that was defined regardless of the history of network. Emerging and non-emerging properties must hence be considered simultaneously. The work we have carried out, and which is still ongoing, is an attempt to build an abstract dynamical analysis of a network where a pre-existing hierarchy influences social exchanges and the distribution of knowledge, but where emerging properties can also be seen as relevant for individual choices of exchange partners.

## 2 Description of the court

Our paper describes some simulations that are led in an artificial society where individuals are organised in a hierarchical system, and have to interact to seek advice. The example that inspired our research is a court that has been studied by two of the authors following a social network analysis approach (Lazega and Mounier, 2003; Lazega, 2003; Falconi and al., 2005). They wanted to witness the evolution of networks within the court, and took as an important indicator the existence of advice seeking among agents. The assumptions on rationality are a mix of theory that existed before (Lazega, 1992) and observations.

The court that is observed, a five century old institution, has been established to settle conflicts that arise in business relations and financial disputes in France. It is composed of judges who are lay judges, actually business people who are related to industries and banks. They have to judge numerous cases every day, some of which are quite complex and require expertise. Most judges stay in this court for 14 years, and their expertise in business law grows over time, in parallel with their hierarchical importance. The official setting of the court is organised through 21 chambers, each having one president, one of them being the president of the whole court. Every year, the chambers are rearranged through a special protocol which is not explained in detail here. One element of the reallocation of judges across chambers is that no chamber contains less than two senior judges (more than 8 years). Indeed, every year, some judges stop their activity, and new comers come in. Only after 8 years of service is it possible to become a chamber president; the title of “president” is still possessed even after this activity has been passed to someone else.

The network study in this court was carried out in two steps separated by two years. A third survey should take place soon so as to constitute a more complete longitudinal survey on the evolution of a network. The characterisation of individuals is made of diverse characteristics defining them within the court (date of entry, chamber, status of president), and others that reveal their social characteristics (age, activity, graduation year). The other indicator that is observed is the existence or inexistence of advice seeking among any two agents. It is considered in general that advice seeking is a sign of strong interrelation among two persons. In the context of a society of peers, it is a sign of trust and respect. In a hierarchical context, it is usually a sign of recognition of the hierarchy. Following the first two surveys led in the court, it was possible to identify regularities in the forms of the relations of individuals.

First, the choice of people who are higher in the *hierarchy* has been shown to be very significant, meaning:

Senior people, who have been in the court for longer time (as the most basic rule, with exceptions that will not be taken into account here)

People who are president of the chamber the judge belongs to

People who are or have been president

People who have an official expertise in law (i.e. with a law degree)

A factor can also represent “hierarchy”, but as an emergent definition: the most central individuals (those who are contacted by the highest number of people) of the court are those who are supposed to be regarded by the group as the most competent, and can thus turn to be important to contact. We consider them as having “high reputation” here.

A second type of choices has been identified: to seek advice from those who have similarity with oneself. For example, those having the same type of activity (especially bankers tend to meet more), those who participate regularly in the same social activities, or those who came into the court on the same year. In the model presented here, we have not used this second set of reasons to describe the rationality of agents, but have only focalised on hierarchy to justify relations.

The way these elements of choice have been abstracted comes from the following reasoning. First the analysts of the advice network have a long knowledge of this setting, and they know the importance of the reproduction of hierarchy for the judges who are part of this court (Lazega and Mounier, 2003). They conducted surveys in which they gathered all advice seeking relations that had occurred among agents so as to reconstitute a complete network. By observing the links and focusing on this notion of hierarchy it was possible to decipher which elements of hierarchy could be relevant (and the fact that judges with a law degree are highly considered then appeared). However, if the common representations and values of the people could be perceived through surveys, no direct inquiry was made on the reason why a specific link

was established. Only its existence was assessed. The issue of rationality being not directly addressed with the judges, some assumptions have been drawn according to previous theories, in coherence with the survey results. The multi-agent system enables us to test the functioning of this simple rationality that could be later combined and see which of these elements have an impact on the shape of the final network. The model is described in the next section.

### 3 Model and simulations

#### 3.1 Court and Agents

The artificial Court contains 147 Agents in 21 Chambers of 7. At the beginning of each year, agents change Chamber. There are at least 2 Agents who have been more than 8 years in the Court in each Chamber. Among these ancient Agents, one is chosen to be the Active President of the Chamber. Any Agent who has been President and is no longer president is a Non-Active President. Each Agent can be the owner of a law degree or not; the quantity of judges with a law degree corresponds to reality: 50%. Every year, Agents who have been in the Court more than 14 years leave and are replaced by new Agents (of age 0).

Each year is divided in 10 successive months. Agents send requests for advice to other agents every month. This is a unilateral communication since we consider that the request is the only relevant indicator in our system. A president (Active or Non-Active) makes twice less requests than a regular Judge. The number of requests made each month is a parameter of the system.

A Judge is defined by: its entry date in the Court; its Chamber; being President (Active or not); having or not having a law degree.

The only individual observation that is made on an Agent is then its centrality during the previous year, which is important for us in different respect as will be seen later. Centrality is the number of requests an Agent received from other Agents (if the same Agent asks three times, it is counted only as one request). The reason why we limit the idea of centrality to the existence or inexistence of at least one request is that in reality we do not have any data on the number of requests.

#### 3.2 Agent's rationality

In our system, Agents always wish to seek advice from those that are higher up in the Hierarchy. As we stated in the previous section, there are two types of hierarchy, one exogenous and the other purely endogenous; and we built from it three types of rationality.

- The purely exogenous one, which we refer to as "authority" rationality: it is based on the difference between President and non-President, and having a law degree vs. not.
- The purely endogenous one which is based on agents' centrality of the previous year, referred to as the "reputation" rationality.
- The mixed rationality is here a very simple mix of both, where Agents choose one or the other rationality alternatively (with 50% chance).

All rationalities are defined through probabilities. For each simulation the probabilities can be changed. The two simple rationalities are here described:

##### Authority rationality:

- o case 1: the Agent never was President (example table 1)
  - chooses its chamber President with probability *probSonPres*
  - otherwise chooses among Presidents (active or non-active) with probabilities *probPres* and among lawyers with *probLaw*

Choice probability	President	Non president
His President	60%	
Law degree	14,4 %	9,6 %
No Law degree	9,6 %	6,4 %

Table 1: choice probability for non-President if  $probHisPres = ProbLaw = ProbPres = 60\%$

- case 2: the Agent is or was President, then chooses with *probPPres* another President and *probPLaw* a judge with a law degree

### Reputation rationality:

On the first time step: chooses randomly among more ancient Agents. Then chooses one of the **NB** most central agents previous year, with probability **Prob** or among the others randomly.

### 3.3 Simulations and observation

A simulation is a succession of 15 years, each made of 10 months. In our system, Presidents make 2 requests for advice and normal Agent make 4. After the system has been initiated, Agents contact each other, leave the Court or get in, as has been explained before. Each Agent starts with an entry date randomly allocated between -13 and -1. At the end none of the Agents was there at the beginning. The simulation is then defined by:

The type of rationality (authority, reputation, mixed, random),

Values of percentages when they are needed:

- *probPres, probHisPres, probPPres, probLaw, probPJur*
- *probRep, nbJugesReputation* lorsque l'agent ne suit que la réputation.

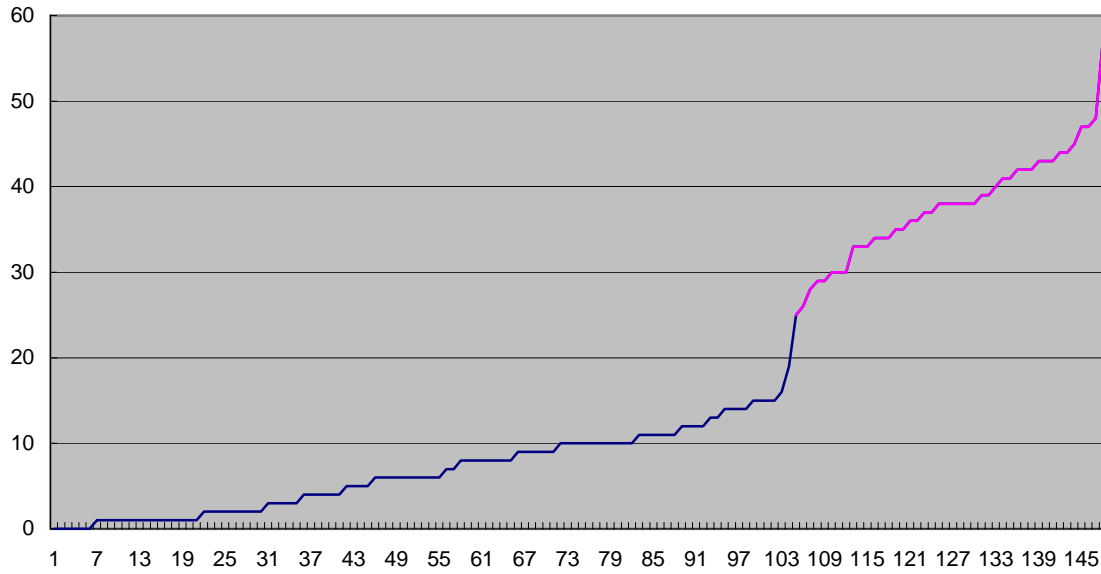
Different data are observed:

- density of links for one year (number of links)
- highest and lowest centrality
- average difference in centrality and mean square deviation
- highest number of demand (several demands can lead to just one link)
- the classification of agents in terms of centrality, which is compared to their status as President and as a judge with a law degree.
- number of links outside a Chamber.

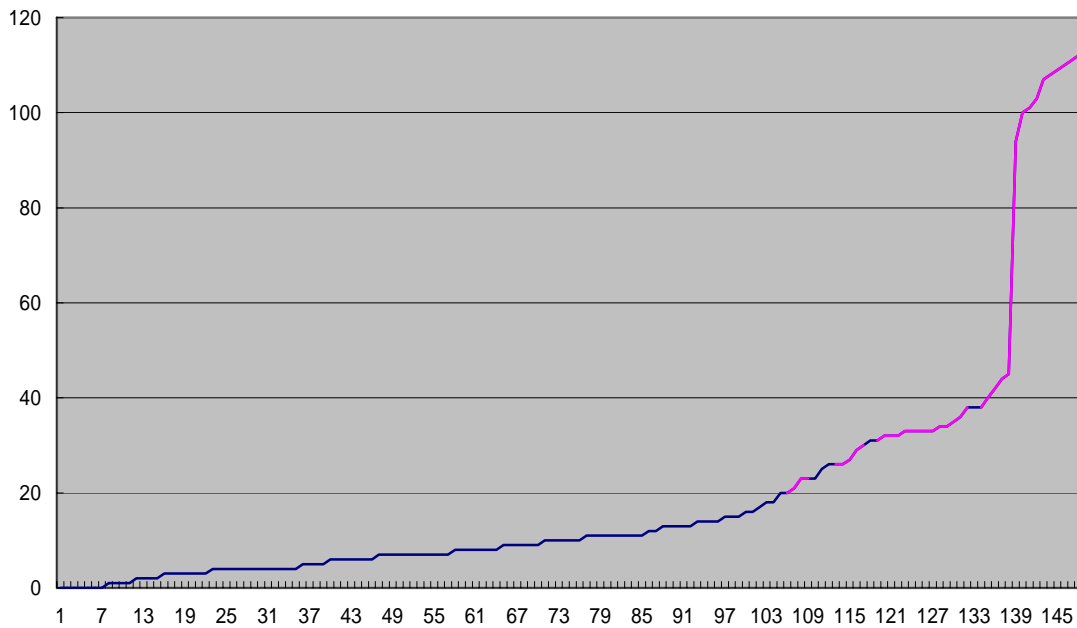
## 4 Results

The way in which centrality is spread among agents very characteristically depends on the type of simulation we observe. In a typical simulation led with just the authority rationality, the link between the centrality of the agent and their quality of President or of a judge with a law degree is very clear. As one can see in figure 1, there are clear gaps between different groups of agents in terms of centrality. Those who belong to the group with higher centrality are generally president of chamber. One element that is not shown in the graph is that the 14 most central all have a law degree here. If one observes Figure 2, one can spot a real difference in the importance of the role of president in simulations where agents have mixed rationality. In this latter case, 12 agents have a centrality of more than 94, where the others are quite equally spread from 0 to 45. Actually the observation of Figure 3 shows how much the mixed rationality is an intermediate between both the other simulations in terms of centrality distribution. It is closer to reputation when it comes to the importance of the gap between the two groups of reputation. In mixed simulation, having a law degree is important for having a high centrality score, but the first agent who does not have a law degree is 9<sup>th</sup> in terms of centrality scores.

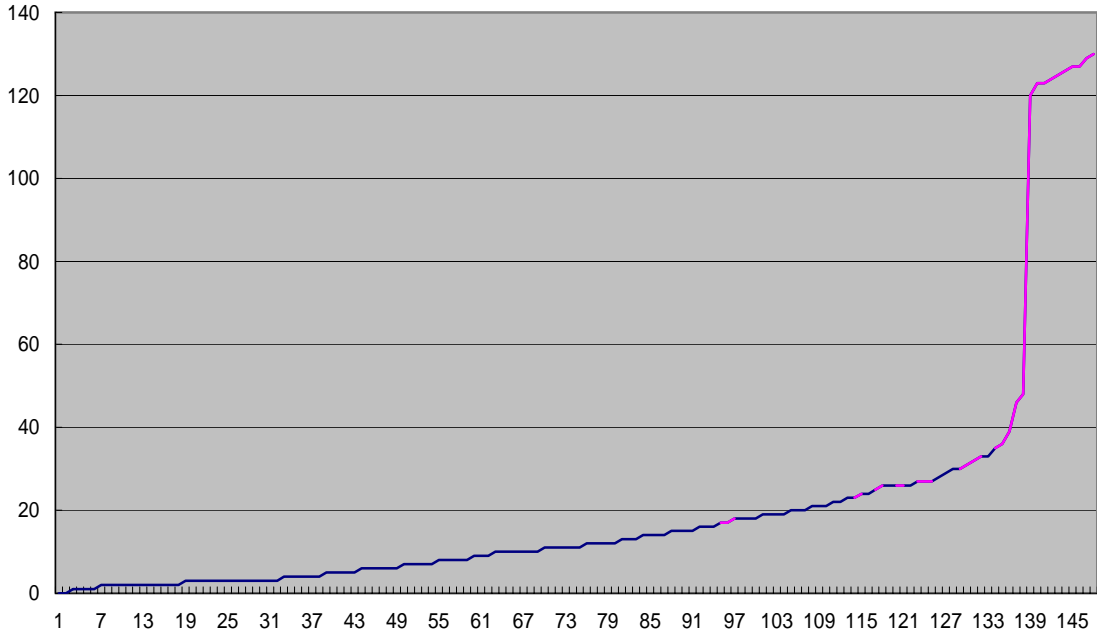
In Reputation simulations, only 10 agents are really separated from the others by a large gap in centrality – the value of centrality for this small group is higher than 120, when the others are lower than 35. In mixed simulations one can see that an intermediate group is present between the higher centrality agents and the others. It is constituted of Presidents: from the use of hierarchy rationality, they get more requests than the non-President Agents, but they get much less than the more central who also get request from the Reputation rationality. Agents in the small group are at the same time central and presidents, and this is why they are much more central than in purely hierarchy simulations.



**Figure 1: Value of centrality for agents at date 15 ordered by increasing value. Rank of the agent is on the x-axis and the value of its centrality on y-axis. Hierarchy rationality (60-60-60). Pink is for Presidents.**



**Figure 2: Value of centrality for agents at date 15 ordered by increasing value. Mixed rationality (60-60-60) (60 - 10). Pink is for Presidents.**



**Figure 3: Value of centrality for agents at date 15 ordered by increasing value. Reputation rationality (60-10). Pink for presidents.**

Adding to this image of the distribution of centrality scores, another type of results is important here (table 2). It is the density: the number of actual links divided by the highest possible number in this network. It shows how structured the system is and the fact that less different links are established (as can be seen with the number of meetings too). Values of density generated in all simulations are very far from those in reality, which is as low as 0.08. The one recorded in hierarchy simulation is the closest to this “real” value, but is still much higher. It means that in reality few people concentrate even more requests for advice. The main reason why we might be unable to fit real data in this context may be due to the fact that in reality judges make more requests, over one year, than the 40 we allow in our simulations.

	Density of relations depending on the type of simulation	Number of meetings per year
Hierarchy 60 60 60	0.105	[2145 – 2312]
Reputation 60 10	0.14	[2932 – 3987]
Mixed	0.13	[2312 – 2991]

**Table 2: Density of relations in different simulations.**

From a dynamical perspective, the indicators in Table 3 show that reputation rationality tends to reinforce an existing emerging order more than the hierarchy logic does, since agents have less chances of losing some centrality over time. Mixed rationality is also somewhat bit better than hierarchy in building a society where being central enables one to become even more central in the future.

	Nb of times centrality decreases	Nb of agent with decreasing centrality
<b>Hierarchy</b>	827	272
<b>Mixed</b>	655	264
<b>Reputation</b>	618	259

**Table 2: Centrality decreases during 15 times-steps in different simulations.**

The results we give here are given only for one value of parameters. The reason why we choose 60 % and 10 most central agents in all these forms of rationality is a choice made by the sociologists who thought it was quite a relevant value. In hierarchy simulations, increasing the chance to meet his own president or a president, a judge with a law degree, gives more structure: the difference is even higher among the groups, but the size of the group with higher centrality stays equivalent.

The influence of increasing the probability of choosing amongst the most central has the same effect in reputation simulation (bigger gap). When this value is decreased, the simulation results then looks increasingly like those of a random simulation, where Agents only choose Agents that were in the Court before them (and which we have no room to describe here). About the other parameter for reputation simulation: the number of agents considered as having high reputation (set to 10 in the simulation presented in Figure 3): if one reduces this number (resp. increases) then the number of agents belonging to the group with high reputation decreases (resp. increases). The gap of centrality that separates both very central and little central Agents is reduced when the number of agents increases.

It is also possible to increase the influence of hierarchy or increase the influence of reputation in mixed simulations by changing the probabilities. The societies produced then resemble more one or the other of the two simple-rationality societies.

## 5 Discussion

In the paper we have for the moment only a partial view of the system we built, but already quite a high level of complexity is represented.

If we consider starting from reputation rationality, the addition of a formal hierarchy is interesting here because it changes the number of people who become central. Instead of having just a very small group that is very separate from the others, much more people can get to be important by answering requests. If we see this result in a sociological perspective, it sheds light on the importance of a formalised hierarchy for the repartition of knowledge and even power. This is an interesting point for our simulation.

If we consider starting from the hierarchical simulation, adding the factor of reputation allows some agents who do not have a law degree to acquire centrality, hence reducing the tautological aspect of the hierarchy logic. This is said to be more realistic by the sociologists.

Our representation of reputation and of rationality in general can be criticised in many ways. For example, there is no feed back on the success of the advice given. Also at no moment have one agents to choose between several agents following a learning process, where a form of judgement over the expertise could be involved. The process where the most contacted tend to become even more contacted over time was criticised. The fact that a highly central person (in real life) would not be able to answer to everyone was an issue for people who listened to the description. These diverse issues are interesting from a purely abstract point of view but not really when it comes to the representation of our case study. Indeed, judges are never judged on their success, since they have more than 97% of success where making choices (a “success” in real life is the fact that the judgement is accepted right away or is confirmed by the Court of Appeal).

If changes have to be made, two can be already envisioned:

- ◆ an improvement of the articulation for mixed rationality: for the moment it is a random behaviour, once out of two times the agent chooses hierarchy or reputation
- ◆ an addition, highly requested by the sociologist: the similarity principle, where the agents tend to go and seek those who are close (or similar) to them (in the same chamber or who came in the same year in the Court). This should radically transform the dynamics of meetings.

The model presented is part of an on-going work, which is in an intermediary position between purely abstract and purely grounded in reality. For the moment we are looking for a dissociation of processes to be able to understand each type of abstract collective learning. Then we will try to assess hypotheses on the importance of hierarchy and emerging network in the case of the actual case study. Our ability to compare results from real life and the model is still limited because our expertise in the



tool is small (the sociologists do not know simulation and the modeller knows little about social network analysis). However the results are already very promising and propose some situations that were not witnessed in the literature until now, which is for us a good first step in this research.

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